Student Number:

2015

YEAR 12

HSC Trial EXAMINATION

Mathematics

General Instructions

- Date of Task – Tuesday 18th August
- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10 on answer sheet provided
- Allow about 15 minutes for this section

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Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

Written by JIO/KLA

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. Which term of the series with *n*th term $T_n = 15 - 2n$ is equal to -37?

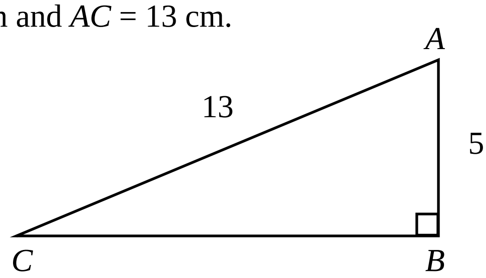
(A) –26

(B) 26

(C) –11

(D) 11

2. The diagram shows the right triangle *ABC*.
$\angle ABC = 90°$, $AB = 5$ cm and $AC = 13$ cm.



What is the value of $\tan \angle BAC$?

(A) $\frac{5}{12}$

(B) $\frac{5}{13}$

(C) $\frac{13}{5}$

(D) $\frac{12}{5}$

3. An infinite geometric series has a first term of 3 and a limiting sum of 1.8.
What is the common ratio?

(A) $-0.\dot{3}$

(B) $-0.\dot{6}$

(C) –1.5

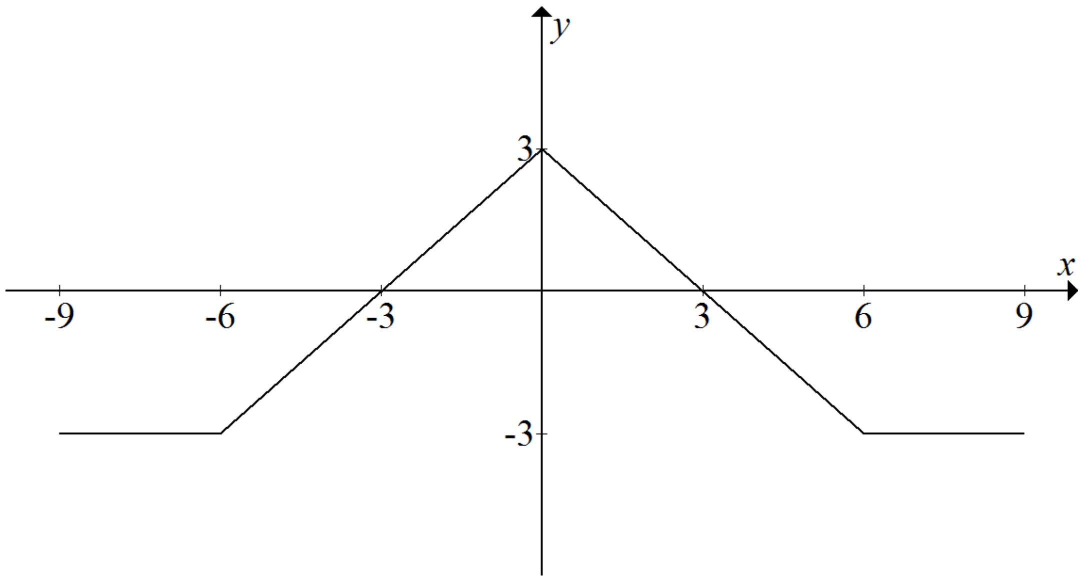
(D) –3.75

4. What is the value of $\int_0^1 (e^{3x} + 1)dx$?

(A) $\frac{1}{3}e^3$ (B) e^3

(C) $\frac{1}{3}(e^3 + 1)$ (D) $\frac{1}{3}(e^3 + 2)$

5. The diagram shows the function $y = f(x)$ in the domain $-9 \le x \le 9$.

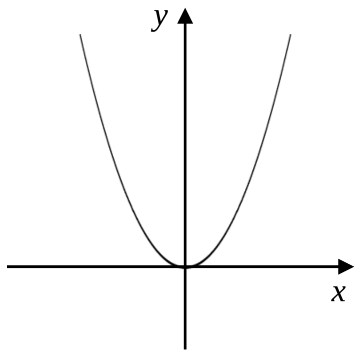


What is the value of $\int_{-9}^{9} f(x)dx$?

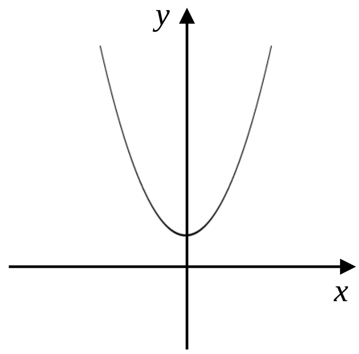
(A) 9
(B) 0
(C) -9
(D) -18

6. Which graph represents a quadratic equation with discriminant $\Delta = 0$?

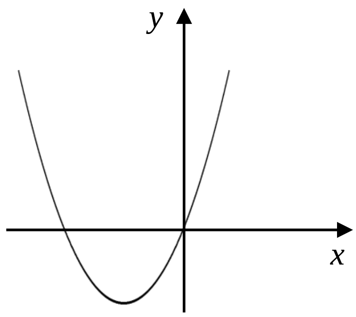
(A)



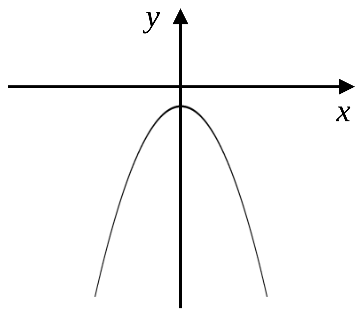
(B)



(C)



(D)



7. What is the solution to the equation $2\cos^2 x - 1 = 0$ in the domain $0 \le x \le 2\pi$?

(A) $x = \frac{\pi}{6}, \frac{11\pi}{6}$

(B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$

(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

8. Which expression is the gradient function of $(5x-4)^7$?

(A) $7(5x-4)^6$

(B) $\frac{5}{8}(5x-4)^8$

(C) $\frac{5}{7}(5x-4)^6$

(D) $35(5x-4)^6$

9. Find the focal length for the parabola $x^2 = 6y + 2x + 11$.

(A) 1

(B) $4a$

(C) 6

(D) $\frac{3}{2}$

10. The equation $x = 3\sin(nt) + 6$ has a period equal to $\frac{3\pi}{4}$.

What is the value of n ?

(A) 2

(B) $\frac{1}{2}$

(C) $\frac{8}{3}$

(D) $\frac{4}{3}$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Show that $\dfrac{1}{\sqrt{2}-1}-\dfrac{1}{\sqrt{2}+1}$ is a rational number **2**

(b) Factorise $8a^3 - y^3$. **2**

(c) Differentiate $\left(x^3-1\right)\left(x^3+1\right)$ **2**

(d) Evaluate $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sqrt{3}\sec^2\frac{x}{3}\,dx$. **3**

(e) Find the sum of the arithmetic series $24 + 28 + 32 + \dots\dots\dots + 136$ **2**

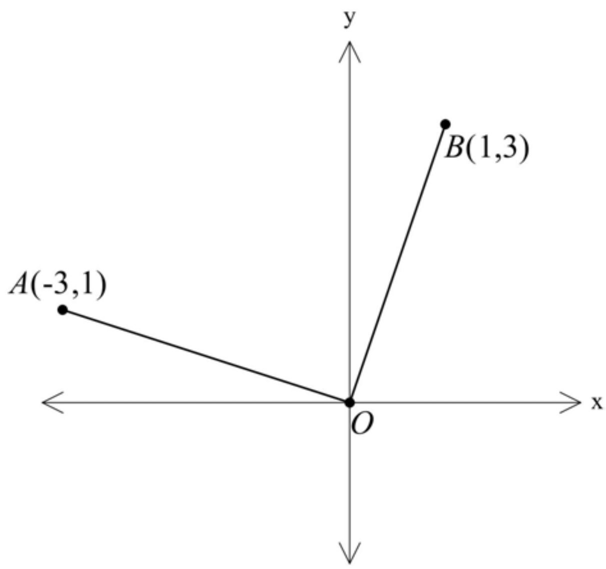
(f) The function $y = ax^3 - x$ has a stationary point at $x = 2$. Find the value of a. **2**

(g) Evaluate $\log_6 9 + \log_6 24$. **2**

End of Question 11

Question 12 (15 marks) **Start a new booklet** **Marks**

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane.



Copy the diagram into your writing booklet.

(i) Find the gradient of line *OA*. **1**

(ii) Show that *OA* is perpendicular to *OB*. **1**

(iii) *OACB* is a quadrilateral in which *BC* is parallel to *OA*. **2**
Show that the equation of *BC* is $x+3y-10=0$.

(iv) The point *C* lies on the line $x=-2$. **1**
What are the coordinates of point *C*?

(v) Show that the length of the line *BC* is $\sqrt{10}$. **1**

(vi) Find the area of *OACB*. **1**

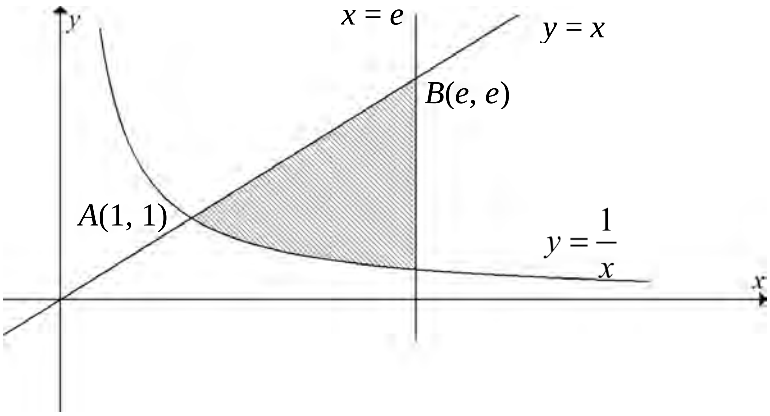
(b) The table shows the values of a function $f(x)$ for five values of *x*. **2**

x	1	1.5	2	2.5	3
$f(x)$	4	1.5	–2	2.5	8

Use Simpson's rule with these five values to estimate $\int_1^3 f(x)dx$.

Question 12 (Continued)

(c) The line $y = x$ and the hyperbola $y = \frac{1}{x}$ intersect at the point $A(1,1)$.
The line $y = x$ and the line $x = e$ intersect at the point $B(e,e)$



Calculate the area enclosed by the line $y = x$, the line $x = e$ and the hyperbola $y = \frac{1}{x}$. **3**

(d) Bag A contains 3 red cubes and 2 white cubes. Bag B contains 2 red cubes and 3 white cubes. A bag is selected at random and then a cube is selected at random from that bag.

(i) Draw a tree diagram to show the possible outcomes. Show the probability on each branch. **2**

(ii) What is the probability that the cube selected is white? **1**

End of Question 12

Question 13 (15 marks) **Start a new booklet** **Marks**

(a) Consider the functions $y = x^2$ and $y = x^2 - 3x + 2$.

(i) Sketch the two functions on the same axes. **2**

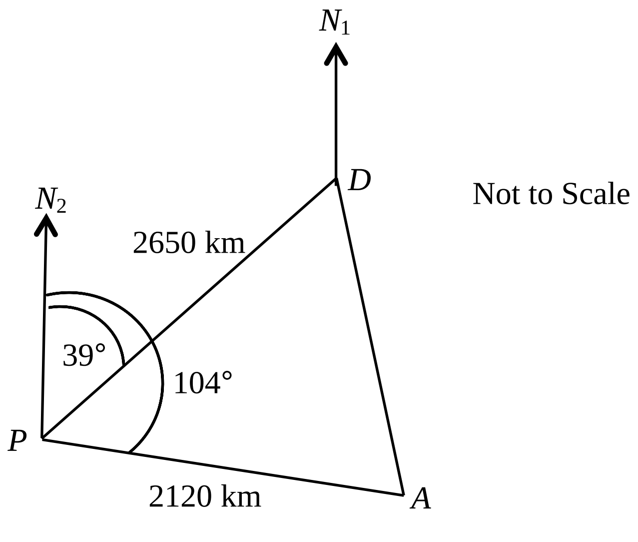
(ii) Hence or otherwise find the values of x such that $x^2 > (x-1)(x-2)$. **1**

(b) Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x)\, dx$ **2**

(c) **(i)** Differentiate $\dfrac{x^2 - 2}{x^2 + 2}$. **2**

(ii) hence evaluate $\displaystyle\int_2^4 \frac{x}{\left(x^2+2\right)^2}\, dx$ **3**

(d) As shown in the diagram below, the bearing of Darwin (D) from Perth (P) is 039°.The distance between the two cities is 2650 km. The bearing of Adelaide (A) from Perth is 104° and the distance between these two cities is 2120 km.



(i) Calculate the distance from Adelaide to Darwin, to the nearest 10 kilometres. **2**

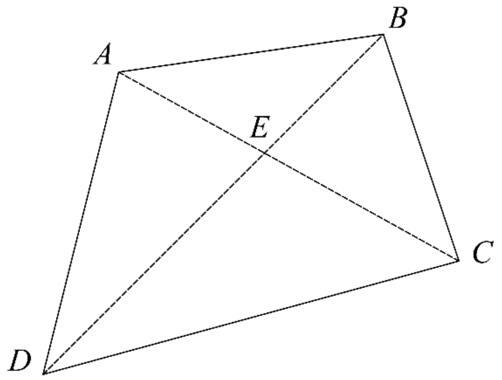
(ii) Find the bearing of Darwin from Adelaide, to the nearest degree. **3**

End of Question 13

Question 14 (15 marks) **Start a new booklet** **Marks**

(a) Differentiate $f(x) = x\cos x$ **1**

(b) In quadrilateral *ABCD* the diagonals *AC* and *BD* intersect at *E*.
Given $AE = 3$, $EC = 6$, $BE = 4$ and $ED = 8$.



Not to scale

(i) Show that $\Delta ABE \parallel\!| \Delta DEC$ **3**

(ii) What type of quadrilateral is *ABCD*? Justify your answer. **2**

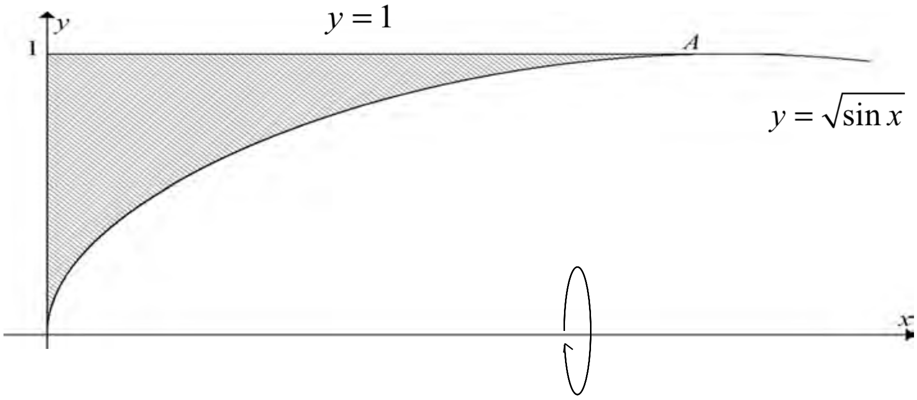
(c) Find the shortest distance between the point $(0,5)$ and the line $3x - y + 1 = 0$ **2**

(d) The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(i) Find the other *x*-intercept of the parabola. **1**

(ii) Find *a*, *b* and *c*. **2**

(e) The region bounded by the curve $y = \sqrt{\sin x}$, the *y*-axis and the line $y = 1$ is rotated around the *x*-axis to form a solid.



(i) If $y = \sqrt{\sin x}$ and $y = 1$ and meet at the point *A*, show that the coordinates of *A* are $\left(\dfrac{\pi}{2}, 1\right)$. **1**

(ii) Find the volume of the solid. **3**

End of Question 14

Question 15 (15 marks) **Start a new booklet** **Marks**

(a) A function $f(x)$ is defined by $f(x) = x^2(3-x)$.

(i) Find the stationary points for the curve $y = f(x)$ and determine their nature. Point(s) of inflexion are **not** required. **3**

(ii) Sketch the graph of $y = f(x)$ showing the stationary points and x-intercepts. **2**

(b) The quadratic equation $2m^2 - 3m + 6 = 0$ has roots α and β.

By considering the sum and the product of the roots, find the value of

$$\alpha^2 + \beta^2$$ **2**

(c) For the first 15 years of his working life, Jonathon puts $1000 at the beginning of each month into a superannuation fund that pays 6% pa interest compounded monthly. For the next 20 years he puts $2000 at the beginning of each month into a superannuation fund that pays 7.5% pa interest compounded monthly.

What is the total value of his superannuation? **3**

(d) The radiation in a rock after a nuclear accident was 8,000 becquerel (bq). One year later, the radiation in the rock was 7,000 bq. It is known that the radiation in the rock is given by the formula:

$$R = R_0 e^{-kt}.$$

where R_0 and k are constants and t is the time measured in years.

(i) Evaluate the constants R_0 and k. **2**

(ii) What is the radiation of the rock after 10 years? Answer correct to the nearest whole number. **1**

(iii) The region will become safe when the radiation of the rock reaches 50 bq. After how many years will the region become safe? **2**

End of Question 15

Question 16 (15 marks) **Start a new booklet** **Marks**

(a) The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term? **3**

(b) The displacement of a particle moving along the *x*-axis is given by

$$x = 5\sin\frac{\pi}{2}t,$$

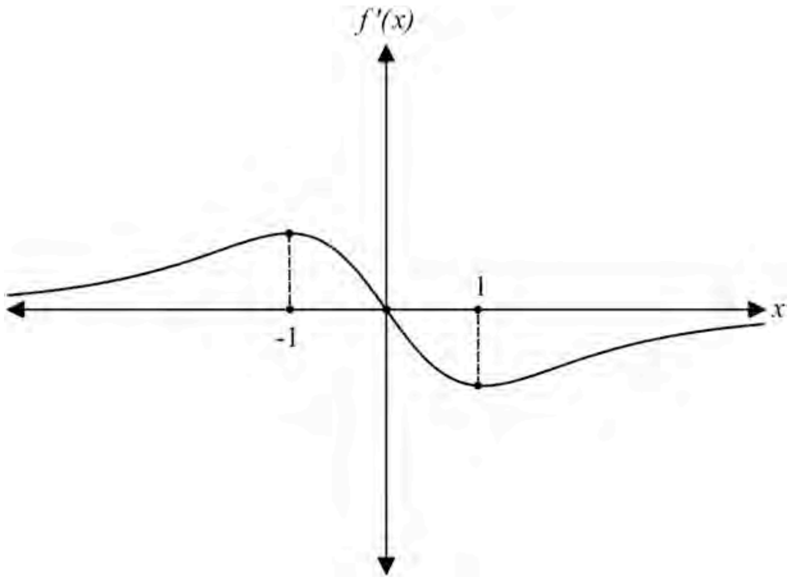
where *x* is the displacement from the origin in metres, *t* is the time in minutes and $t \geq 0$.

(i) What is the furthest distance the particle moves away from the origin. **1**

(ii) When does the particle first return to its starting position? **1**

(iii) Find the acceleration of the particle when $t = 3\text{min}$. **3**

(c) The graph of $f'(x)$ shown in the diagram passes through the origin. **3**
As $x \to \pm\infty$ $f'(x) \to 0$ and $f(x) \to 0$.



Sketch the graph of $y = f(x)$, given $f(x) > 0$.

Question 16 (Continued)

(d) A triangle *ABC* is right-angled at *C*. *D* is the point on *AB* such that *CD* is perpendicular to *AB*. Let $\angle BAC = \theta$.

Draw a diagram showing this information.

(i) Given that $8AD + 2BC = 7AB$, show that **2**

$$8\cos\theta + 2\tan\theta = 7\sec\theta$$

(ii) Hence or otherwise, find θ **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Student number:

Trial HSC Examination, 2015
Mathematics

Multiple Choice Answer Sheet for Section 1

Completely colour in the response oval representing the most correct answer.

1	A	⬭	B	⬭	C	⬭	D	⬭
2	A	⬭	B	⬭	C	⬭	D	⬭
3	A	⬭	B	⬭	C	⬭	D	⬭
4	A	⬭	B	⬭	C	⬭	D	⬭
5	A	⬭	B	⬭	C	⬭	D	⬭
6	A	⬭	B	⬭	C	⬭	D	⬭
7	A	⬭	B	⬭	C	⬭	D	⬭
8	A	⬭	B	⬭	C	⬭	D	⬭
9	A	⬭	B	⬭	C	⬭	D	⬭
10	A	⬭	B	⬭	C	⬭	D	⬭

Mark: **/10**

YR 12 MATHEMATICS TRIAL HSC 2015

SOLUTIONS

SECTION I (10 marks)

1.
$$15 - 2n = -37$$
$$-2n = -52$$
$$n = 26$$
B

2.
$$CB = \sqrt{13^2 - 5^2} = 12$$
$$\tan \angle BAC = \frac{12}{5}$$
D

3.
$$1.8 = \frac{3}{1-r}$$
$$1.8 - 1.8r = 3$$
$$-1.8r = 1.2$$
$$r = -\frac{12}{18} = -\frac{2}{3} = -0.\dot{6}$$
B

4.
$$\left[\frac{1}{3}e^{3x} + x\right]_0^1$$
$$= \left(\frac{1}{3}e^3 + 1\right) - \left(\frac{1}{3} + 0\right)$$
$$= \frac{1}{3}e^3 + 1 - \frac{1}{3}$$
$$= \frac{1}{3}e^3 + \frac{2}{3}$$
$$= \frac{1}{3}(e^2 + 2)$$
D

5.
$$-(3\times3) + -(3\times3)$$
$$= -9 - 9$$
$$= -18$$
D

6. **A**

7.
$$2\cos^2 x = 1$$
$$\cos^2 x = \frac{1}{2}$$
$$\cos x = \pm\frac{1}{\sqrt{2}}$$
$$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$
D

8.
$$y = (5x - 4)^7$$
$$\frac{dy}{dx} = 7(5x-4)^6 \cdot 5$$
$$= 35(5x-4)^6$$
D

9.
$$x^2 - 2x = 6y + 11$$
$$x^2 - 2x + 1 = 6y + 12$$
$$(x-1)^2 = 6(y+2)$$

Vertex $(1, -2)$

Focal length: $4a = 6$
$$a = \frac{6}{4} = 1\frac{1}{2}$$
$\therefore$ Focus $\left(1, -\frac{1}{2}\right)$ **D**

10.
$$\frac{2\pi}{n} = \frac{3\pi}{4}$$
$$3n = 8$$
$$n = \frac{8}{3}$$
C

QUESTION 11 (15 MARKS)

(a) $\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} - \left(\frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} \right)$

$= \frac{\sqrt{2}+1}{1} - \left(\frac{\sqrt{2}-1}{1} \right)$ 1

$= \sqrt{2}+1 - \sqrt{2} + 1$ 1

$= 2$

(b) $(2a)^3 - (y)^3$ 1

$= (2a-y)(4a^2 + 2ay + y^2)$ 1

(c) $(x^3-1)(x^3+1)$

$= x^6 - 1$ 1

$\frac{d}{dx}(x^6 - 1) = 6x^5$ 1

(d) $\left[3\sqrt{3} \tan \frac{x}{3} \right]_{\frac{\pi}{2}}^{\pi}$ 1

$= 3\sqrt{3}\left[\tan \frac{\pi}{3} - \tan \frac{\pi}{6} \right]$

$= 3\sqrt{3}\left[\sqrt{3} - \frac{1}{\sqrt{3}} \right]$ 1

$= 3 \times 3 - 3$

$= 6$ 1

(e) Find n

$a = 24 \quad d = 4 \quad T_n = 136$

$24 + 4(n-1) = 136$

$24 + 4n - 4 = 136$

$4n = 116$

$n = 29$ 1

$S_{29} = \frac{29}{2}(24 + 136)$

$= 2320$ 1

(f) $\frac{dy}{dx} = 3ax^2 - 1 = 0$ 1

at $x = 2$ $\quad 3a \times 2^2 - 1 = 0$

$12a - 1 = 0$

$12a = 1$

$a = \frac{1}{12}$ 1

(g) $\log_6 (9 \times 24)$

$= \log_6 216$ 1

$6^x = 216$

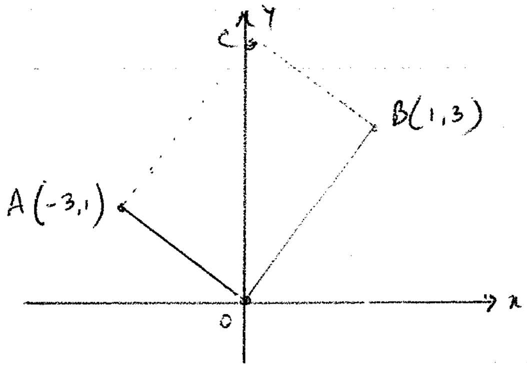
$\therefore x = 3$ 1

(OR just use change of base

$\frac{\log_{10} 216}{\log_{10} 6} = 3$)

QUESTION 12 (15 MARKS)

(a)



(i) Grad OA $= -\frac{1}{3}$ 1

(ii) Grad OB $= \frac{3}{1} = 3$ 1

$3 \times -\frac{1}{3} = -1 \quad \therefore OA \perp OB$

(iii) grad $= -\frac{1}{3}$ pt $(1, 3)$

$y - 3 = -\frac{1}{3}(x - 1)$ 1

$3y - 9 = -x + 1$ 1

$x + 3y - 10 = 0$ Eqn of BC

(iv) Sub $x = -2$ into $x + 3y - 10 = 0$

$-2 + 3y - 10 = 0$

$3y = 12$

$y = 4$

$\therefore$ Pt C is $(-2, 4)$ 1

(v) $d_{BC} = \sqrt{(-2-1)^2 + (4-3)^2}$

$= \sqrt{10}$ units 1

(vi) $BC = \sqrt{10}$ units

$\therefore$ AREA OACB $= \sqrt{10} \times \sqrt{10}$ 1

$= 10$ sq. units

(b) $A = \frac{2-1}{6}\{f(1) + 4f(1.5) + f(2)\}$

$+ \frac{3-2}{6}\{f(2) + 4f(2.5) + f(3)\}$ 1

$= \frac{1}{6}[4 + 4 \times 1.5 + (-2) + -2 + 4 \times 2.5 + 8]$

$= \frac{1}{6}(8 + 16)$

$\doteqdot 4$ 1

(c) $A = \int_1^e x \, dx - \int_1^e \frac{1}{x} \, dx$ 1

$= \left[\frac{x^2}{2} - \ln x\right]_1^e$ 1

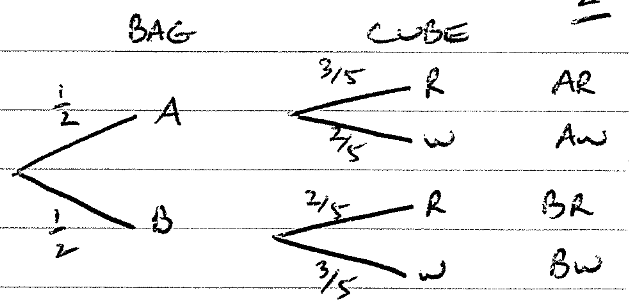
$= \left(\frac{e^2}{2} - 1\right) - \left(\frac{1}{2} - 0\right)$

$= \frac{e^2}{2} - 1 - \frac{1}{2}$

$= \frac{e^2}{2} - \frac{3}{2}$ 1

$= \frac{1}{2}(e^2 - 3)$ sq. units

(d) 2



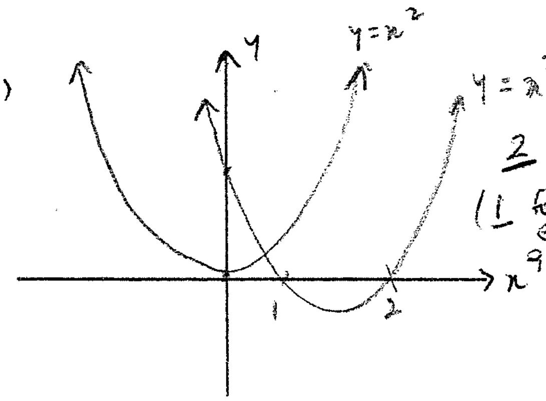
$P(\text{white}) = P(Aw) + P(Bw)$

$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5}$

$= \frac{5}{10} = \frac{1}{2}$ 1

QUESTION 13 (15 MARKS)

(a) (i)



2 (1 for each graph)

(ii) $x^2 > x^2-3x+2$

$0 > -3x+2$

$3x > 2$

$\therefore \quad x > \frac{2}{3}$ 1

(b) $\left[\frac{x^3}{3} - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{\pi^3}{648} - \frac{1}{2}\cos\frac{\pi}{3}\right) - \left(0 - \frac{1}{2}\right)$

$= \frac{\pi^3}{648} - \frac{1}{4} + \frac{1}{2}$

$= \frac{\pi^3}{648} + \frac{1}{4} \qquad (0.297....)$

(c) (i) $y = \frac{x^2-2}{x^2+2}$

$\frac{dy}{dx} = \frac{(x^2+2).2x - (x^2-2).2x}{(x^2+2)^2}$ 1

$= \frac{2x^3+4x-2x^3+4x}{(x^2+2)^2}$

$= \frac{8x}{(x^2+2)^2}$ 1

(c) (ii) $\int_2^4 \frac{x}{(x^2+2)^2}\,dx$

$= \left[\frac{1}{8}\left(\frac{x^2-2}{x^2+2}\right)\right]_2^4$ 1

$= \frac{1}{8}\left(\frac{4^2-2}{4^2+2} - \frac{2^2-2}{2^2+2}\right)$ 1

$= \frac{1}{8}\left(\frac{14}{18} - \frac{2}{6}\right)$

$= \frac{1}{8}\times\frac{4}{9} = \frac{1}{18}$ 1

(d) (i) $\angle DPA = 104^\circ - 39^\circ$

$= 65^\circ$ 1

$AD^2 = 2650^2 + 2120^2 - 2.2650.2120.\cos 65^\circ$

$= 6768361.211...$

$A = 2601.607.$ 1

$\doteqdot 2600$ km (nearest 10 km)

(ii) $\frac{\sin\angle PDA}{2120} = \frac{\sin 65^\circ}{2600}$

$\sin\angle PDA = \frac{\sin 65^\circ}{2600}\times 2120$ 1

$= 0.738989426..$

$\angle DAP = 48^\circ$ (nearest degree)

$\angle N_1DP = 180^\circ - 39^\circ = 141^\circ$ 1

$\therefore \angle N_1DA = 360^\circ - 141^\circ - 48^\circ$

$= 171^\circ$

$\therefore$ Bearing of Darwin from Adelaide

$= 360^\circ - (180^\circ - 171) = 351^\circ$ 1

(4)

QUESTION 14 (15 MARKS)

(a) $f(x) = x\cos x$

$f'(x) = x.-\sin x + \cos x . 1$

$= \cos x - x\sin x$ 1

(b)(i) In $\triangle ABE$ and $\triangle DEC$,

$\angle AEB = \angle DEC$ (vert opp $\angle$s are equal) 1

$\frac{AE}{EC} = \frac{3}{6} = \frac{1}{2}$ $\frac{BE}{ED} = \frac{4}{8} = \frac{1}{2}$ 1

$\therefore \triangle ABE ||| \triangle DEC$

(two pairs of corresponding sides are in proportion and the included angles are equal) 1

(ii) $\angle BAE = \angle DCE$ (matching $\angle$s in similar triangles are equal)

$\therefore$ $\angle BAE$ and $\angle DCE$ are alternate angles and equal.

$\therefore AB \parallel CD$ (alternate angles are only equal if the lines $\parallel$)

$\therefore$ ABCD is a trapezium

(one pair opposite sides parallel)

(c) $(\overset{x_1}{0}, \overset{y_1}{5})$ $3x - y + 1 = 0$

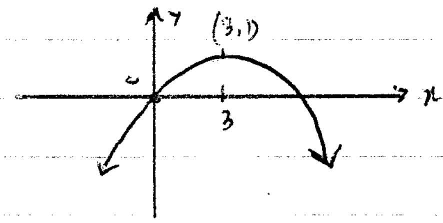
$a = 3$ $b = -1$ $c = 1$

$d = \frac{|0.3 + 5.-1 + 1|}{\sqrt{3^2 + (-1)^2}}$ 1

$= \frac{|-4|}{\sqrt{10}} = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$ 1

(d) (i) Parabola is symmetrical about the vertex (3, 1)



1

$\therefore$ Other x-intercept is (6, 0)

(ii) All three points (0,0), (3,1), (6,0) satisfy $y = ax^2 + bx + c$

So, sub points into $y = ax^2 + bx + c$ and create 3 equations 1

(0,0) $0 = 0 + 0 + c$

$\therefore c = 0$

(3,1) $1 = 9a + 3b$ — ①

(6,0) $0 = 36a + 6b$ — ②

Solve ① and ② simultaneously

① × 2 $2 = 18a + 6b$ — ③

$0 = 36a + 6b$ — ②

② − ③ $-2 = 18a$

$a = -\frac{1}{9}$

Sub into ① $1 = 9 \times -\frac{1}{9} + 3b$

$1 = -1 + 3b$

$b = \frac{2}{3}$

$\therefore a = -\frac{1}{9}$, $b = \frac{2}{3}$ and $c = 0$.

1

(e) PTO

14 continued

(e) (i) $\sqrt{\sin x} = 1$

$\sin x = 1$

$\therefore x = \frac{\pi}{2}$

Coordinates are $\left(\frac{\pi}{2}, 1\right)$ 1

(ii) $V = \pi \int_1^{\frac{\pi}{2}} y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{2}} 1 \, dx - \pi \int_0^{\frac{\pi}{2}} \sin x \, dx$ 1

$= \pi \left[x\right]_0^{\frac{\pi}{2}} + \pi \left[\cos x\right]_0^{\frac{\pi}{2}}$

$= \pi . \left(\frac{\pi}{2} - 0\right) + \pi \left(\cos \frac{\pi}{2} - \cos 0\right)$ 1

$= \frac{\pi^2}{2} + \pi(0 - 1)$

$= \frac{\pi^2}{2} - \pi$ cu. units 1

or $\left(\pi\left(\frac{\pi}{2} - 1\right) \text{ cu units} \right)$

QUESTION 15 (15 MARKS)

(a) $f(x) = x^2(3-x)$

x-intercepts are $x=0$, $x=3$

$f(x) = 3x^2 - x^3$

$f'(x) = 6x - 3x^2$

$f''(x) = 6 - 6x$ 1

(i)

St. pt occur when $f'(x)=0$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0$ $x = 2$

$(0,0)$ $(2,4)$ 1

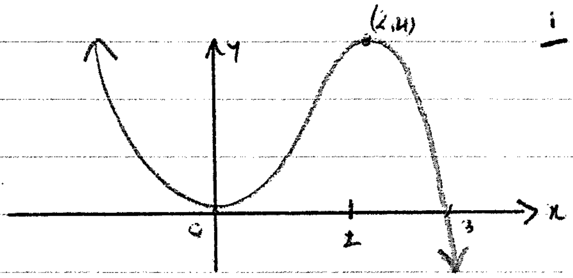
Test:

$f''(0) = 6 > 0$ $f''(2) = 6-12 = -6 < 0$

$\therefore$ Min turn. pt at $(0,0)$ $\therefore$ MAX at $(2,4)$

1

(ii)



(b) $\alpha + \beta = \frac{--3}{2} = 1\frac{1}{2}$

$\alpha\beta = \frac{6}{2} = 3$ 1

$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{3}{2})^2 - 2\times 3$

$= \frac{9}{4} - 6$ 1

$= -\frac{15}{4} = -3\frac{3}{4}$

(c) For First 15 years

$P = \$1000$ $r = 6\%$ p.a $= 0.005$ per month $n = 15\times 12 = 180$ months

$A = 1000(1.005)^{180} + 1000(1.005)^{179} + \dots + 1000(1.005)^2 + 1000(1.005)$

$= 1000[1.005 + 1.005^2 + \dots + 1.005^{180}]$

$= 1000\left[\frac{1.005(1.005^{180}-1)}{1.005-1}\right]$ ✱

$a = 1.005$ $r = 1.005$ $n = 180$

For next 20 years

$P = \$2000$ $r = 7.5\%$ p.a $= 0.00625$ per month $n = 20\times 12 = 240$ months

$A = 2000(1.00625)^{240} + 2000(1.00625)^{239} + \dots + 2000(1.00625)^2 + 2000(1.00625)^1$

$= 2000[1.00625 + 1.00625^2 + \dots + 1.00625^{240}]$

$= 2000\left[\frac{1.00625(1.00625^{240}-1)}{1.00625-1}\right]$ ✱

$a = 1.00625$ $r = 1.00625$ $n = 240$

So, total the two amounts

Total = ✱ + ✱

$= \$292272.806 + \$1114383.084$

$= 1406655.89$

$= \$1406656$ 3

15 continued

(d) (i) Initially $t=0$, $R_0 = 8000$ 1

$R = R_0 e^{-kt}$

$R = 8000 e^{-kt}$

Also, when $t=1$, $R = 7000$

$7000 = 8000 e^{-k \times 1}$

$7000 = 8000 e^{-k}$

$e^{-k} = \frac{7}{8}$

$\ln e^{-k} = \ln \frac{7}{8}$ 1

$-k = \ln \frac{7}{8}$

$\therefore k = -\ln(\frac{7}{8}) = 0.13353139...$

(ii) when $t = 10$

$R = 8000 e^{-0.13353... \times 10}$

$= 2104.604...$

$= 2105$ kg. 1

(iii) Find t when $R = 50$

$50 = 8000 e^{-0.13353... \times t}$

$e^{-0.13353... \times t} = \frac{5}{800}$ 1

$-0.13353... \times t = \ln\left(\frac{1}{160}\right)$

$t = \ln\left(\frac{1}{160}\right) \div -0.13353...$

$= 38.0073458...$

$= 38$ years. 1

(8)

QUESTION 16 (15 MARKS)

(a) $ar^6 = 20$ — ①

$ar^2 = 1.25$ — ② 1

$1 \div 2$

$r^4 = 16$

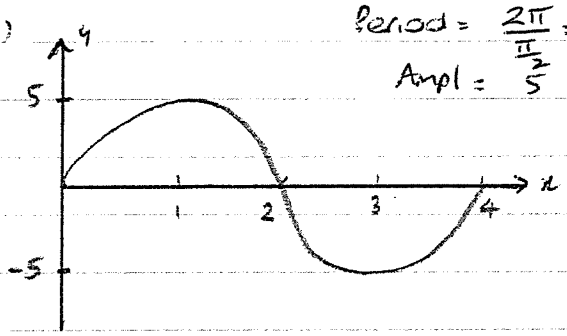
$r = \pm 2$ 1

Sub into ② $a \times (2)^2 = 1.25$

$a = \frac{5}{16}$ 1

b) (i) Period $= \frac{2\pi}{\frac{\pi}{2}} = 4$

Ampl $= 5$



Furthest distance = 5 m 1

(ii) $t = 2$ minutes 1

(iii) $v = \dot{x} = \frac{5\pi}{2} \cos \frac{\pi}{2} t$ 1

$a = \ddot{x} = -\frac{5\pi^2}{4} \sin \frac{\pi}{2} t$ 1

when $t = 3$

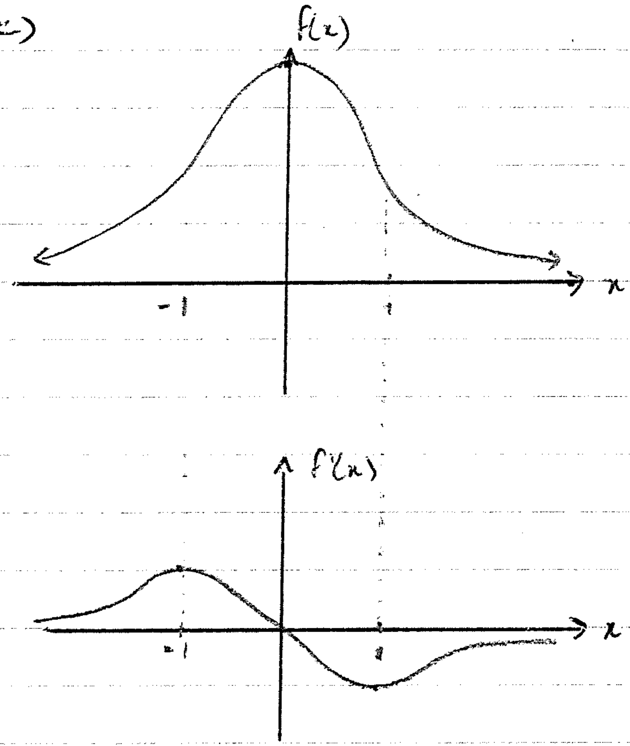
acceleration

$= -\frac{5\pi^2}{4} \times \sin \frac{3\pi}{2}$

$= \frac{5\pi^2}{4}$ m / min² 1

(12.337.... m/min²)

(c)



1 shows max turning pt at $x = 0$

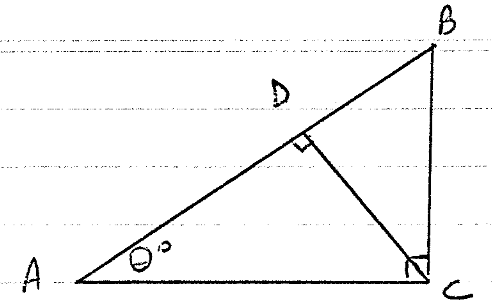
1 correct shape

1 correct position of pts of inflexion ~~at x = -1 and x = 1~~

(d) P.T.O

16 (c)

(i)



$\cos\theta = \frac{AD}{AC}$ $\quad\quad$ $\tan\theta = \frac{BC}{AC}$ $\quad\quad$ $\cos\theta = \frac{AC}{AB}$ $\quad$ 1

$AD = AC\cos\theta$ $\quad\quad$ $BC = AC\tan\theta$ $\quad\quad$ $AB = AC \times \frac{1}{\cos\theta}$

$= AC\sec\theta$

Now $\quad 8AD + 2BC = 7AB$ $\quad$ 1

$8AC\cos\theta + 2AC\tan\theta = 7AC\sec\theta$

$\div AC \quad \therefore 8\cos\theta + 2\tan\theta = 7\sec\theta$

(ii) $\quad 8\cos\theta + 2\tan\theta = 7\sec\theta$

$8\cos\theta + \frac{2\sin\theta}{\cos\theta} = \frac{7}{\cos\theta}$

$8\cos^2\theta + 2\sin\theta = 7$

$8(1-\sin^2\theta) + 2\sin\theta = 7$ $\quad$ 1

$8\sin^2\theta - 2\sin\theta - 1 = 0$

Let $u = \sin\theta$ $\quad 8u^2 - 2u - 1 = 0$

$(2u-1)(4u+1) = 0$

$u = \frac{1}{2}$ $\quad$ or $\quad$ $u = -\frac{1}{4}$

$\sin\theta = \frac{1}{2}$ $\quad\quad$ $\sin\theta = -\frac{1}{4}$

$\theta = 30°$ $\quad\quad$ $\theta = 165° 31'$

Since $0° \le \theta \le 90°$ (θ is in a right angled triangle) $\quad$ 1

$\therefore \theta = 30°$

(10)